CHAPTER 3

Symmetrical Fault Analysis

3.1 Introduction

The fault analysis of a power system is required to provide information for the selection of switchgear, setting of relays and stability of system operation. A power system is not static but changes during the operation (switching on or off of generators and transmission lines) and planning (addition of generators and transmission lines). Thus fault studies need to be routinely performed by the utility engineers.

Faults usually occur in a power system due to

- insulation failure of equipment.
- flashover of lines initiated by a lighting stroke.
- permanent damage to conductors and towers or accidental faulty operations.

Faults may either be three-phases in nature involving all three-phases in a symmetrical manner, or may be asymmetrical where usually only one or two phases may be involved. Faults may also be caused either by short-circuits to earth, between live conductors, or by broken conductors in one or more phases. Sometimes simultaneous faults may occur involving both short-circuit and broken conductor faults (also known as open-circuit fault).

3.2 Types of Faults

(i) Series fault or open-circuit fault
   - One open conductor fault
   - Two open conductor fault

(ii) Shunt fault or short-circuit fault
   - Symmetrical fault or balanced fault
     — Three-phase fault

- Unsymmetrical fault or unbalanced fault
  - Line-to-ground (L-G) fault
  - Line-to-line (L-L) fault
  - Double line-to-ground (L-L-G) fault

A three-phase fault is a condition where either (a) all the three-phases of the system are short-circuited to each other, or (b) all the three-phases of the system are earthed. This type of fault is defined as the simultaneous short-circuited fault which occurs at all the three-phases and gives rise to symmetrical current. It occurs infrequently, but it is the most severe type. This fault current is determined by the internal emf of the machine in the system, the internal impedances and the impedance in the network between machine and fault.

The balanced three-phase faults may be analysed using per phase basis analysis or equivalent single-phase circuit. With asymmetrical three-phase faults, the use of symmetrical components helps to reduce the complexity of the calculations as transmission lines and components are by and large symmetrical, although the fault may be asymmetrical.

Fault analysis is usually carried out in per-unit quantities as they give solutions which are somewhat consistent over different voltage and power ratings, and operate on values of the order of unity.

This, in general, is a balanced condition, and we just need to know the positive-sequence network to analyse faults. Further, the single line diagram can be used, as all the three-phases carry equal currents displaced by 120°.

Typically, only 5% of the initial faults in a power system are three-phase faults with or without earth. Of the unbalanced faults, 80% are line-earth and 15% are double line faults with or without earth and which can often deteriorate to three-phase fault. Broken conductor faults account for the rest.

A fault represents a structural network change equivalent with that caused by the addition of impedance at the place of a fault. If the fault impedance is zero, the fault is referred to as bolted fault or solid fault.

3.3 Transient (Short-circuit) on Power System Components

A modern large interconnected power system components have inductive property which gives rise to transients when there is a sudden change in current.

3.3.1 Transient (Short-circuit) on a Transmission Line

Let us consider the short-circuit transient on a transmission line. Certain simplifying assumptions are made at this stage. The line is fed from the constant voltage source. Short-circuit takes place when the line is unloaded and the line capacitance is negligible.

Consider the series *R–L* circuit as shown in Figure 3.1. The closing of switch (SW) at $t = 0$ represents a first approximation of a three-phase short-circuit at the terminals of an unloaded transmission line. The current is assumed to be zero before switch closes, and the source angle α determines the source voltage at $t = 0$. Now the Kirchhoff's voltage law equation for the circuit

$$Ri(t) + L\frac{di(t)}{dt} = \sqrt{2}V \sin(\omega t + \alpha) \qquad t \geq 0 \tag{3.1}$$

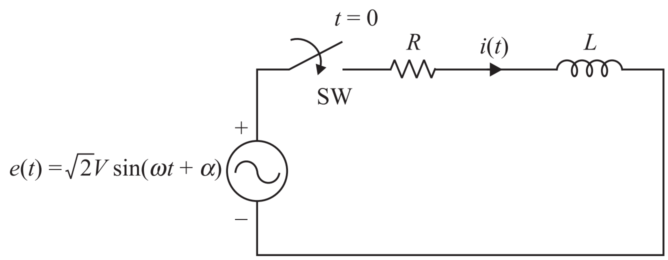


Figure 3.1 Simple series *R–L* circuit.

The solution to the above equation is as follows.

Total short-circuit current

$$i(t) = i_{ac}(t) + i_{dc}(t)$$

$$i(t) = \frac{\sqrt{2}V}{|Z|}\sin(\omega t + \alpha - \theta) + \frac{\sqrt{2}V}{|Z|}\sin(\theta - \alpha)\, e^{-(R/L)t} \tag{3.2}$$

where symmetrical short-circuit current,

$$i_{ac}(t) = \frac{\sqrt{2}V}{|Z|}\sin(\omega t + \alpha - \theta) \tag{3.3}$$

dc offset current,

$$i_{dc}(t) = \frac{\sqrt{2}V}{|Z|}\sin(\theta - \alpha)\, e^{-(R/L)t} \tag{3.4}$$

Impedance of the transmission line

$$Z = \sqrt{R^2 + (\omega L)^2} \tag{3.5}$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \tag{3.6}$$

The total short-circuit current in Eq. (3.2) is plotted in Figure 3.2 along with two components. The symmetrical short-circuit current given by Eq. (3.3) is a sinusoidal. The dc offset current, given by Eq. (3.4), decays exponentially with time constant.

The total short-circuit current $i(t)$, the value corresponding to the first peak, is called maximum momentary short-circuit current i_{mm}. If the decay of the transient current in this short time is neglected, then

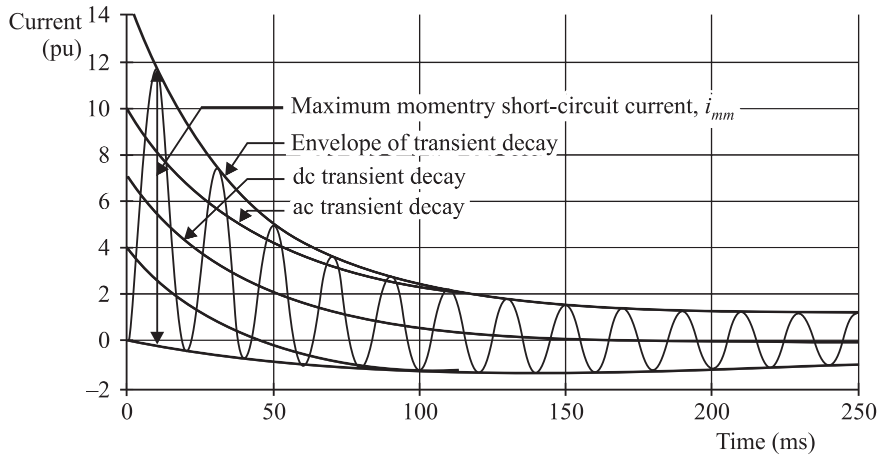


Figure 3.2 Waveform of a short-circuit current on a transmission line.

$$i_{mm} = \frac{\sqrt{2}V}{|Z|}\sin(\theta - \alpha) + \frac{\sqrt{2}V}{|Z|} \tag{3.7}$$

Since the transmission line resistance is small, $\theta \cong 90°$.

Therefore,
$$i_{mm} = \frac{\sqrt{2}V}{|Z|}\cos\alpha + \frac{\sqrt{2}V}{|Z|} \tag{3.8}$$

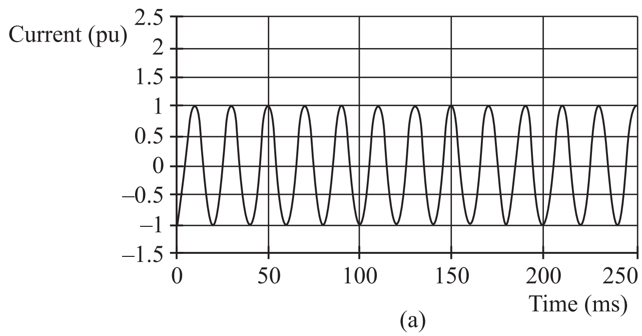
From Eq. (3.8), i_{mm} has the maximum possible value when $\alpha = 0$. This implies that the effect of short-circuit will be severe if the fault occurs when the voltage wave is going through zero. Thus

$$i_{mm\,(\text{max possible})} = \frac{\sqrt{2}V}{|Z|} + \frac{\sqrt{2}V}{|Z|} = \frac{2\sqrt{2}V}{|Z|} = \text{doubling effect} \tag{3.9}$$

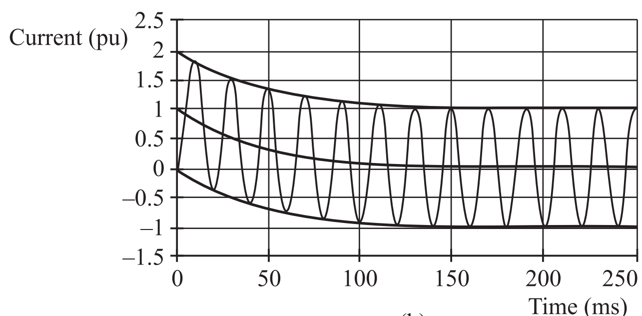
For the selection of circuit breakers, the momentary short-circuit current is taken corresponding to its maximum possible value.

3.3.2 Transient (Short-circuit) on a Synchronous Machine

As mentioned earlier, the current flowing in the power system network during a fault depends on the machines connected to the system. Due to the effect of the armature current on the flux that generates the voltage, the current flowing in a synchronous machine differs immediately after the occurrence of the fault, a few cycles later, and under sustained or steady state conditions. Further there is an exponentially decaying dc component caused by the instantaneous value at the instant of fault occurring. These are shown in Figure 3.3. Figures 3.3(a) and 3.3(b) depict the steady state current waveform and the transient waveform of a simple *R–L* circuit to show the decay in the dc component.



(a)



(b)

Figure 3.3 Steady state and transient waveform of transmission line.

In addition to this, in the synchronous machine, the magnitude of the ac current peak also changes with time as shown in Figure 3.4, with the unidirectional component of the transient waveform removed. Due to the initial low back emf at the instant of fault resulting in high current, the effective impedance is very low. Even when the dc transient component is not present, the initial current can be several times the steady state value.

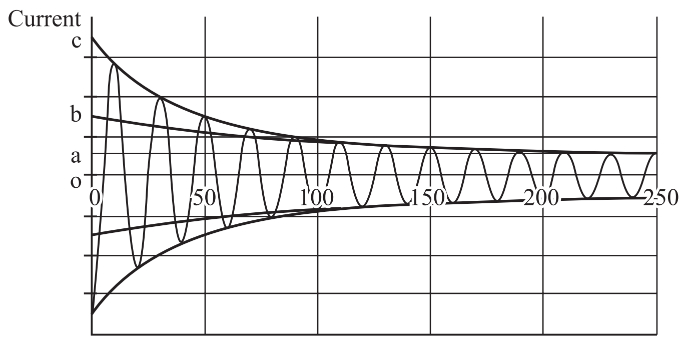


Figure 3.4 Symmetrical short-circuit armature current in synchronous machine.

Thus the three regions are identified for determining the reactance. These are the subtransient reactance X_d'' for the first 10 to 20 ms of fault, the transient reactance X_d' for up to about 500 ms, and the steady state reactance X_d (synchronous reactance).

Under the steady state three-phase short-circuit condition, the armature reaction of an alternator produces a demagnetizing flux. This effect is represented as a reactance called armature reaction reactance, X_a. The combined of armature reactance, X_a and leakage reactance, X_l is called synchronous reactance, X_s. In case of a salient pole alternator, the synchronous reactance is called direct axis reactance, X_d.

Direct axis subtransient reactance (X_d''): At the instant of short-circuit, the dc offset current appears in all the three-phases of stator. This dc offset current can induce current in rotor field winding and damper winding by the transformer action. The increase in field current and damper winding current will set up flux in a direction to augment the main flux. This effect can be represented by two reactances in parallel with X_a as shown in Figure 3.5. Here X_f represents the flux created by induced current in the field winding and X_{dw} indicates the flux created by induced current in the damper winding. The combined effect of all the three reactances is to reduce the total reactance of the machine and so the short-circuit current is very high in this state which is called subtransient state. That is, the total reactance under this condition is subtransient reactance.

$$X_d'' = X_l + \cfrac{1}{\cfrac{1}{X_a} + \cfrac{1}{X_f} + \cfrac{1}{X_{dw}}} \tag{3.10}$$

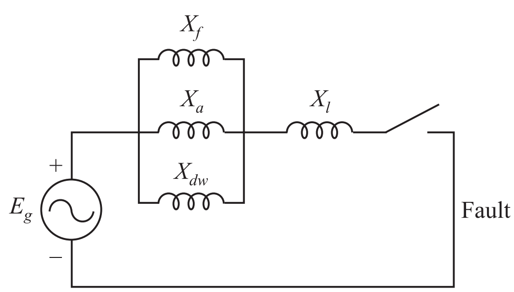


Figure 3.5 Circuit model of subtransient reactance.

Direct axis transient reactance (X_d'): The reactance is effective after the damper winding currents have died out, i.e. the transient reactance of the machine as shown in Figure 3.6 is given by

$$X_d' = X_l + (X_a \parallel X_f)$$

or

$$X_d' = X_l + \cfrac{1}{\cfrac{1}{X_a} + \cfrac{1}{X_f}} \tag{3.11}$$

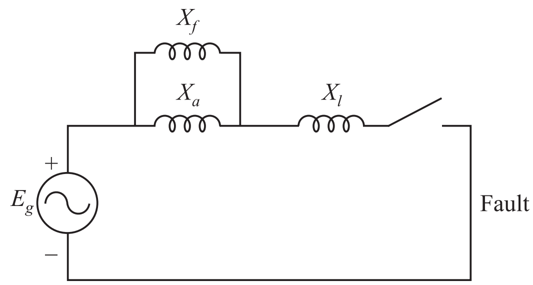


Figure 3.6 Circuit model of transient reactance.

Direct axis synchronous reactance or steady state condition reactance: The transient state will exist for a few cycles and then the steady state conditions are achieved as the effect of field winding current will also die out in short time depending on its time constant. Thus the steady state total reactance as shown in Figure 3.7 is given by the sum of X_a and X_l.

$$X_d = X_a + X_l \tag{3.12}$$

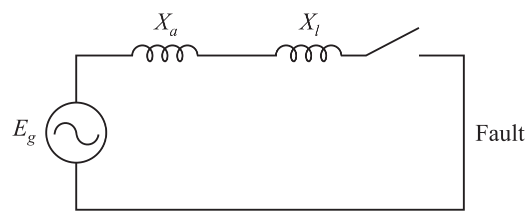


Figure 3.7 Circuit model of steady state reactance.

The fundamental frequency component of armature current following the sudden application of short-circuit to the armature of an initially unloaded machine can be expressed as

$$i_{ac}(t) = \sqrt{2}E_g\left[\left(\frac{1}{X''_d} - \frac{1}{X'_d}\right)e^{-t/\tau''_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d}\right)e^{-t/\tau'_d} + \frac{1}{X_d}\right]\sin\left(\omega t + \alpha - \frac{\pi}{2}\right) \tag{3.13}$$

where E_g is the rms line to lone neutral pre-fault terminal voltage of the unloaded synchronous machine. The armature resistance is neglected in the above equation.

Note that at time $t = 0$, when the fault occurs the rms value of current

$$i_{ac}(0) = I'' = \frac{E_g}{X''_d} \tag{3.14}$$

which is called the rms subtransient fault current, I''. The duration of I'' is determined by the time constant τ''_d, which is called the direct axis short-circuit subtransient time constant.

At a later time, when t is large compared to τ''_d, but small compared to the direct axis short-circuit transient time constant τ'_d, the first exponential term in Eq. (3.13) has decayed almost to zero, but the second exponential has not decayed significantly. The rms ac fault current then equals to the rms transient fault current and given by

$$I' = \frac{E_g}{X'_d} \tag{3.15}$$

when t is much larger than τ'_d, the rms ac fault current approaches its steady state value, given by

$$I = \frac{E_g}{X_d} \tag{3.16}$$

After a fault occurs, the subtransient, transient, and steady state periods are characterized by the subtransients reactance X''_d, the transient reactance X'_d and steady state reactance X_d respectively. These reactances have increasing values ($X''_d < X'_d < X_d$) and the corresponding components of the short-circuit current have decreasing magnitudes ($|I''| > |I'| > |I|$). With dc component removed, the initial symmetrical rms current is the rms value of the ac component of the fault current immediately after the fault occurs.

Internal voltages of loaded machines under fault conditions

Let us consider a generator that is loaded when a fault occurs. Figure 3.8 shows the equivalent circuit of generator that has a balanced three-phase load.

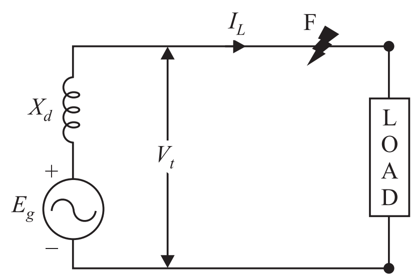


Figure 3.8 Equivalent circuit of a loaded generator under steady state condition.

I_L is the current delivered by the generator. The circuit model of a synchronous generator operating under steady state condition supplying a load current I_L is shown in Figure 3.8.

$$E_g = V_t + jI_LX_d \tag{3.17}$$

where E_g is the induced emf under loaded condition

X_d is the direct axis synchronous reactance of the machine.

V_t is the terminal voltage of the generator.

If a three-phase fault or short-circuit occurs at point F, we see that a short-circuit from F to neutral in the equivalent circuit does not satisfy the conditions for calculating subtransient current, for the reactance of the generator.

Now to study the subtransient state, E_g and X_d of Figure 3.8 should be replaced by E_g'' and X_d'' as shown in Figure 3.9.

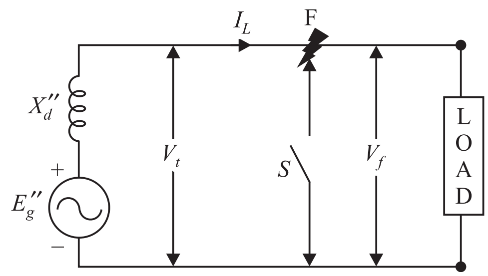


Figure 3.9 Equivalent circuit of a loaded generator under subtransient condition.

$$E_g'' = V_t + jI_L X_d'' = V_f + jI_L X_d'' \tag{3.18}$$

where E_g'' is the subtransient internal voltage.

In order to study the transient state, E_g and X_d of Figure 3.8 should be replaced by E_g' and X_d' as shown in Figure 3.10.

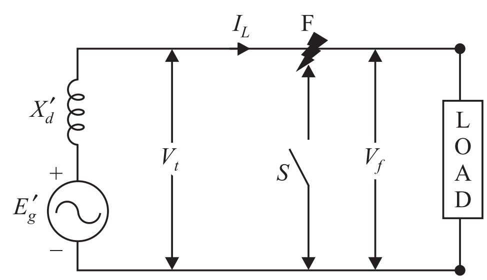


Figure 3.10 Equivalent circuit of a loaded generator transient condition.

$$E_g' = V_t + jI_L X_d' = V_f + jI_L X_d' \tag{3.19}$$

where E_g' is the transient internal voltage.

The **synchronous motors** have internal emfs and reactances similar to that of generator except that the current direction is reversed.

$$E_g'' = V_t - jI_L X_d'' \tag{3.20}$$

$$E_g' = V_t - jI_L X_d' \tag{3.21}$$

3.4 Symmetrical Short-circuit Current Calculation Through Thevenin's Theorem

An alternative method of computing short-circuit current is through the Thevenin's theorem. This method is faster and easily adapted to systematic computation for large networks.

Consider a synchronous generator feeding a synchronous motor over a transmission line and the fault occurs at motor terminals as shown in Figure 3.11. The fault current and the bus voltage and the line current during the fault can be determined and the fault voltage and current can be obtained by using the prefault voltage and current.

3.4.1 Procedure for Symmetrical Short-circuit Current Calculation Through Thevenin's Theorem

1. Assume all prefault voltage magnitudes are 1.0 per unit and all prefault currents are zero.
2. Draw the single line diagram (Figure 3.11).

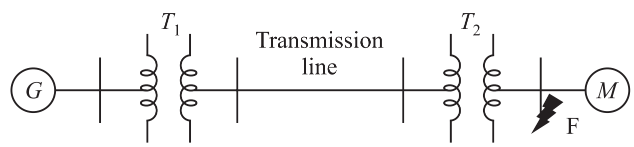


Figure 3.11 Single line diagram of representative power system.

3. Draw the reactance diagram. The per unit reactance values are determined from the per unit analysis (Figures 3.12 and 3.13).

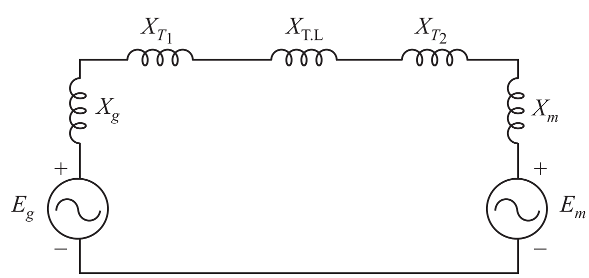


Figure 3.12 Reactance diagram of power system.

4. Prefault bus voltage and current are obtained from the result of load flow analysis.
5. Replace the reactances of synchronous machines by their subtransient/ transient values.
6. Short-circuited all the emf sources. The result is the passive Thevenin's network (Figure 3.13).

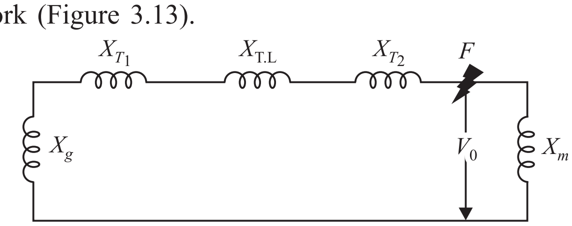


Figure 3.13 Reactance diagram of power system.

7. Draw Thevenin's equivalent circuit viewed from the faulted bus.
8. Find the fault current

$$I_f = \frac{E_{th}}{j(X_{th} + X_f)} \tag{3.22}$$

where, E_{th} or $V°$ is the prefault voltage
Thevenin reactance, $X_{th} = (X_g + X) \parallel X_m$
X_f is the fault reactance.

$$X = X_{T1} + X_{TL} + X_{T2}$$

9. Determine the current contributed by the generator and motor, etc.

$$I_G = \frac{jX_m}{j(X_g + X + X_m)} I_f\,; \qquad I_M = \frac{j(X_g + X)}{j(X_g + X + X_m)} I_f \tag{3.23}$$

10. Determine the postfault voltage using

$$V_{i(f)} = V_{i(0)} + \Delta V = V_{i(0)} + (-jX_{th}I_f) \tag{3.24}$$

11. Determine the postfault line flows

$$I_{ij(f)} = \frac{(V_{i(f)} - V_{j(f)})}{z_{ij}} \tag{3.25}$$

3.4.2 Short-circuit Capacity (SCC) or Short-circuit MVA or Fault Level Calculations

In a power system, the maximum fault current (or fault MVA) that can flow into a zero impedance fault is necessary to be known for switch gear solution. This can either be the balanced three-phase value or the value at an asymmetrical condition. The fault level defines the value for the symmetrical condition. The fault level is usually expressed in MVA (or the corresponding per unit value), with the maximum fault current value being converted using the nominal voltage rating.

The short-circuit capacity (SCC) is defined by the product of magnitude of prefault bus voltage and postfault current.

$$\text{Short-circuit capacity} = |E_{th}| \times |I_f| \tag{3.26}$$

$$\text{Short-circuit capacity} = E_{th} \times \frac{E_{th}}{Z_{th}} \qquad (Z_f = 0) = \frac{E_{th}^2}{Z_{th}}$$

The per unit voltage for nominal value is unity, so that

$$\text{Fault level (p.u.)} = \frac{1}{Z_{th}} \tag{3.27}$$

$$\text{Fault MVA} = \text{fault level (p.u.)} \times \text{MVA}_{\text{base}} = \frac{\text{MVA}_{\text{base}}}{Z_{th}} \tag{3.28}$$

$$\text{Base current} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\ \text{kV}_\text{b}} \times 10^3 \tag{3.29}$$

$$\text{Fault current in } A = \text{fault current in p.u. } (I_f) \times \text{base current} \tag{3.30}$$

The SCC of a busbar is the fault level of the busbar. The strength of a busbar (or the ability to maintain its voltage) is directly proportional to its SCC. An infinitely strong bus (or infinite busbar) has an infinite SCC, with zero equivalent impedance and will maintain its voltage under all conditions.

The magnitude of short-circuit current is time dependant due to synchronous generators. It is initially at its largest value and decreasing to steady value. These higher fault levels tax circuit breakers (CBs) adversely so that current limiting reactors can be used.

The short-circuit MVA is a better indicator of the stress on CBs than the short-circuit current as CB has to withstand recovery voltage across breaker following arc interruption. The current flowing during a fault is determined by the internal emfs of machines in the network, the impedances of the machines, and the impedances between the machines and the fault.

3.5 Selection of Circuit Breaker

The circuit breakers are protective devices which are used in power system to automatically open the faulty part of the system in the event of a fault. In normal working condition they can be used as a switch. Hence the two functions of CBs are as follows:

- To act as switch for normal load conditions
- To automatically isolate the faulty part in the event of a fault

Two of the CB ratings which require the computation of SC current are:

1. Rated momentary current and
2. Rated symmetrical interrupting current.

Symmetrical short-circuit current is obtained by using subtransient reactance for synchronous machines. Momentary current (rms) is then calculated by multiplying the symmetrical momentary current by a factor of 1.6 to account for the presence of dc offset current.

The CB for a particular application is selected on the basis of the following ratings.

1. Normal working power level specified as rated interrupting current or rated interrupting kVA.
2. The fault level specified as either the rated short-circuit interrupting current or rated short-circuit current interrupting MVA.
3. Momentary current rating
4. Normal working voltage
5. Speed of CB

The symmetrical current to be interrupted is computed by using subtransient reactances for synchronous generators and transient reactances for synchronous motors. The dc offset value to be added to obtain the current to be interrupted is accounted for by multiplying the symmetrical SC current by a factor as tabulated below.

Speed of CB	*Multiplying factor*
8 cycles or more	1.0
5 cycles	1.1
3 cycles	1.2
2 cycles	1.4
1½ cycles	1.5

$$\text{Short-circuit current interrupting MVA} = \sqrt{3} \times |V_{pf\,L}| \times |I_{f\,L}| \tag{3.31}$$

$$\text{Short-circuit current interrupting in p.u} = \sqrt{3} \times |V_{pf\,\text{p.u.}}| \times |I_{f\,\text{p.u.}}| \times \text{MVA}_b \tag{3.32}$$

EXAMPLE 3.1 Generators G_1 and G_2 are identical and rated 11 kV, 20 MVA and have a transient reactance of 0.25 p.u. at own MVA base. The transformers T_1 and T_2 are also identical and are rated 11/66 kV, 5 MVA and have a reactance of 0.06 p.u. to their own MVA base. A 50 km long transmission line is connected between the two generators. Calculate the three-phase fault current, when fault occurs at the middle of the line as shown in Figure 3.14.

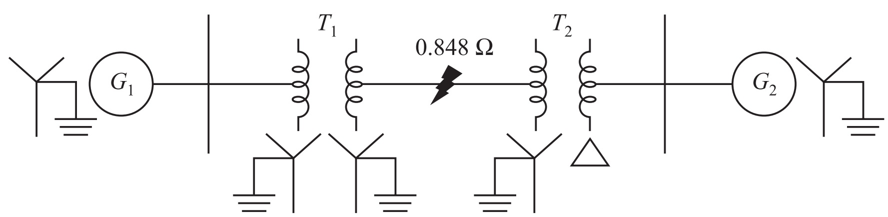


Figure 3.14 Single line diagram.

Solution:

$$\text{Base MVA, MVA}_{\text{new}} = 20 \text{ MVA}$$
$$\text{Base kV, kV}_{\text{new}} = 11 \text{ kV}$$

Reactance of generator G_1

$X_{\text{p.u.(given)}} = 0.25$ p.u., $\text{MVA}_{\text{given}} = 20$, $\text{MVA}_{\text{new}} = 20$, $\text{kV}_{\text{given}} = 11$, $\text{kV}_{\text{new}} = 11$

$$X_{\text{p.u.(new)}} = 0.25 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{20}{20}\right) = 0.25 \text{ p.u.}$$

Reactance of transformer T_1 (Primary side)

$X_{\text{p.u.(given)}} = 0.06$ p.u., $\text{MVA}_{\text{given}} = 5$, $\text{MVA}_{\text{new}} = 20$, $\text{kV}_{\text{given}} = 11$, $\text{kV}_{\text{new}} = 11$

$$X_{\text{p.u.(given)}} = j0.06 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{20}{5}\right) = j0.24 \text{ p.u.}$$

Reactance of transmission line

Middle of the line, (25 km long), actual reactance = 0.848 × 25 = 21.2 Ω

Base kV on HT side of transformer T_1

$$= \text{base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

$$\text{Base kV on HT s ide of transformer } T_1 = 11 \times \frac{66}{11} = 66 \text{ kV}$$

$$\text{kV}_{\text{new}} = 66 \text{ kV}$$

$$\text{Base impedance} = \frac{(\text{kV}_{\text{new}})^2}{\text{MVA}_{\text{new}}} = \frac{66^2}{20} = 217.8\ \Omega$$

Per unit reactance of the transmission line

$$= \frac{\text{actual reactance, }\Omega}{\text{base reactance, }\Omega} = \frac{21.2}{217.8} = j0.0973 \text{ p.u.}$$

Reactance of transformer T_2 (Primary side)

$X_{\text{p.u.(given)}} = 0.06$ p.u., $\text{MVA}_{\text{given}} = 5$, $\text{MVA}_{\text{new}} = 20$, $\text{kV}_{\text{given}} = 11$ $\text{kV}_{\text{new}} = 11$

$$X_{\text{p.u.(given)}} = j0.06 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{20}{5}\right) = j0.24 \text{ p.u.}$$

Reactance of generator G_2

$X_{\text{p.u.(given)}} = 0.25$ p.u., $\text{MVA}_{\text{given}} = 20$, $\text{MVA}_{\text{new}} = 20$, $\text{kV}_{\text{given}} = 11$, $\text{kV}_{\text{new}} = ?$

Base kV on LT side of transformer T_2

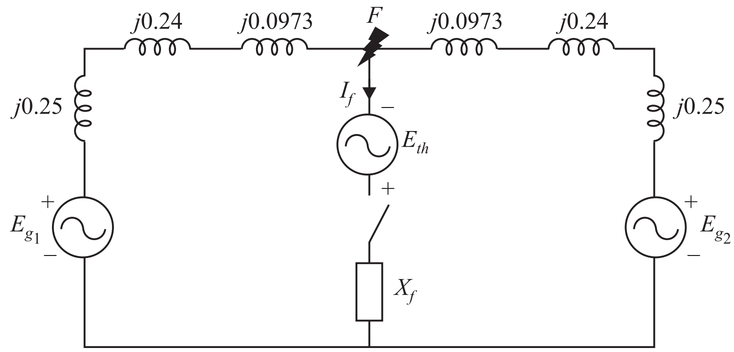
$$= \text{base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$\text{Base kV on LT side of transformer } T_2 = 66 \times \frac{11}{66} = 11 \text{ kV}$$

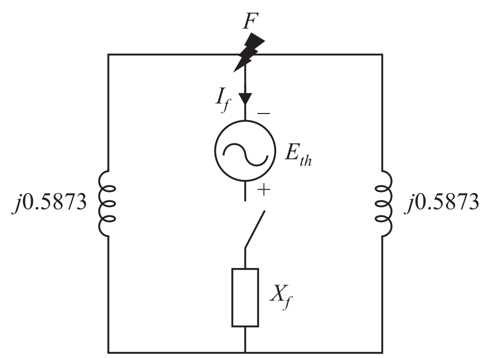
$$\text{kV}_{\text{new}} = 11 \text{ kV}$$

$$X_{\text{p.u.(new)}} = 0.25 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{20}{20}\right) = j0.25 \text{ p.u.}$$

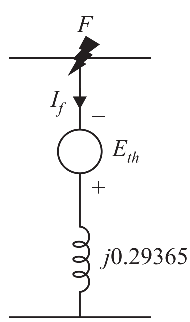
Prefault reactance diagram of Example 3.1



The above diagram is reduced to $j0.25 + j0.24 + j0.0973 = j0.5873$.



Thevenin equivalent network of Example 3.1



Thevenin equivalent impedance, X_{th} is $j0.5873 \parallel j0.5873$

$$\therefore \qquad Z_{th} \text{ or } X_{th} = \frac{j0.5873 \times j0.5873}{j0.5873 + j0.5873} = j0.29365 \text{ p.u.}$$

Prefault voltage or Thevenin voltage, $E_{th} = 1\angle 0°$

Fault reactance or impedance, $X_f = 0$

Fault current

$$I_f = \frac{E_{th}}{j(X_{th} + X_f)} = \frac{1\angle 0°}{j0.29365} = -\,j3.405 \text{ p.u.}$$

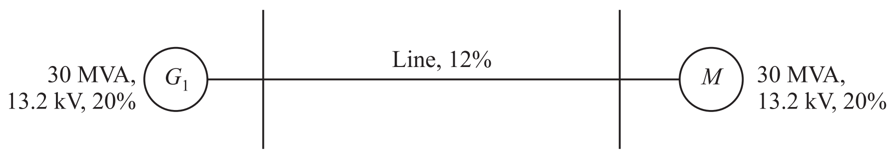
$$\text{Base current} = \frac{\text{MVA}_{\text{Base}}}{\sqrt{3}\ \text{kV}_\text{b}} \times 10^3 = \frac{20}{\sqrt{3} \times 11} \times 10^3 = 1049.73 \text{ A}$$

Fault current in A = fault current in p.u. (I_f) × base current

Fault current in A, $|I_f|$ = 3.405 × 1049.73 = 3574.32 A

EXAMPLE 3.2 A synchronous generator and synchronous motor each rated 30 MVA, 13.2 kV and both have subtransient reactance of 20% and the line reactance of 12% on a base of machine ratings. The motor is drawing 25 MW at 0.85 p.f. leading. The terminal voltage is 12 kV when a three-phase short-circuit fault occurs at motor terminals. Determine the subtransient current in generator, motor and at the fault point.

Solution: Single line diagram



Base MVA, MVA_{new} = 30 MVA
Base kV, kV_{new} = 13.2 kV

Reactance of generator G_1

$X_{\text{p.u.(given)}}$ = 0.2 p.u., $\text{MVA}_{\text{given}}$ = 30, MVA_{new} = 30, kV_{given} = 13.2
kV_{new} = 13.2

$$X_{\text{p.u.(new)}} = 0.2 \times \left(\frac{13.2}{13.2}\right)^2 \times \left(\frac{30}{30}\right) = 0.2 \text{ p.u.}$$

Reactance of transmission line

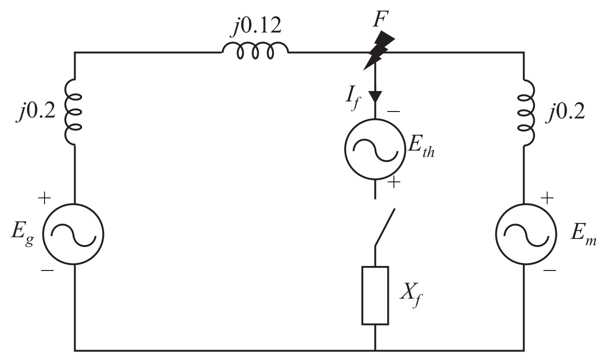
Actual reactance = 12% = 0.12 Ω

Reactance of motor M

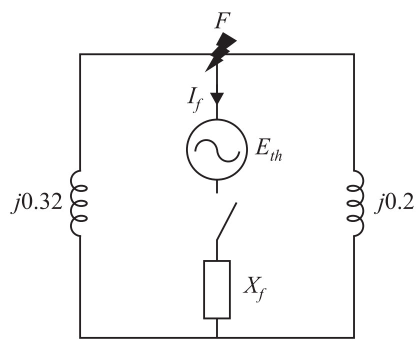
$X_{\text{p.u.(given)}}$ = 0.2 p.u., $\text{MVA}_{\text{given}}$ = 30, MVA_{new} = 30, kV_{given} = 13.2
kV_{new} = 13.2

$$X_{\text{p.u.(new)}} = 0.2 \times \left(\frac{13.2}{13.2}\right)^2 \times \left(\frac{30}{30}\right) = 0.2 \text{ p.u.}$$

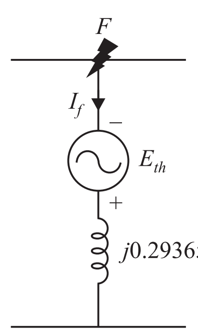
Prefault reactance diagram of Example 3.2



The above diagram is reduced to $j0.2 + j0.12 = j0.32$



Thevenin equivalent network of Example 3.2



Thevenin equivalent impedance, X_{th} is $j0.32 \parallel j0.2$

$$\therefore \qquad Z_{th} \quad \text{or} \quad X_{th} = \frac{j0.32 \times j0.2}{j0.32 + j0.2} = j0.1231 \text{ p.u.}$$

Actual prefault voltage at fault point = 12 V

Base kV, kV_{new} = 13.2 kV

Per unit prefault voltage or Thevenin voltage, $E_{th} = \dfrac{12}{13.2} = 0.9091\angle 0°$

Fault reactance or impedance, $X_f = 0$

(i) *Subtransient fault current*

$$I_f = \frac{E_{th}}{j(X_{th} + X_f)} = \frac{0.9091\angle 0^\circ}{j0.1231} = \frac{0.9091\angle 0^\circ}{0.1231\angle 90^\circ} = 7.385\angle{-90^\circ} \text{ p.u.}$$

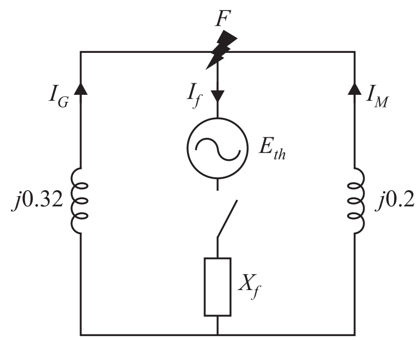
$$\text{Base current} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\text{kV}_b} \times 10^3 = \frac{30}{\sqrt{3} \times 13.2} \times 10^3 = 1312.16 \text{ A}$$

Fault current in kA = fault current in p.u.(I_f) × base current

Fault current in kA $|I_f| = 7.385\angle{-90^\circ} \times 1312.16$

$= 9.690\angle{-90^\circ}$ kA

(ii) *Subtransient fault current contributed by generator and motor*



$$I_G = \frac{jX_m}{j(X_g + X + X_m)} I_f = \frac{j0.2}{j0.52} \times 7.385\angle{-90^\circ} = 2.840\angle{-90^\circ}$$

$$I_M = \frac{j(X_g + X)}{j(X_g + X + X_m)} I_f = \frac{j0.32}{j0.52} \times 7.385\angle{-90^\circ} = 4.545\angle{-90^\circ}$$

EXAMPLE 3.3 The two-bus system is shown in Figure 3.15. Determine the total three-phase fault current and the fault current supplied by each generator at the faulted point.

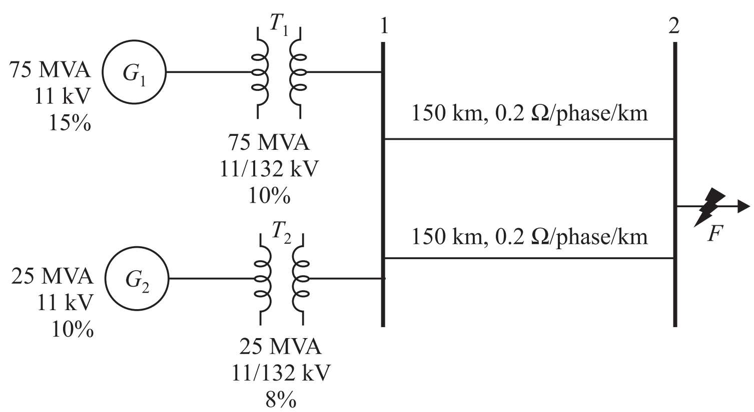


Figure 3.15 Single line diagram.

Solution:

Base MVA, MVA_{new} = 75 MVA
Base kV, kV_{new} = 11 kV

Reactance of generator G_1

$X_{p.u.(given)} = 0.15$ p.u., $MVA_{given} = 75$, $MVA_{new} = 75$, $kV_{given} = 11$, $kV_{new} = 11$

$$X_{p.u.(new)} = 0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{75}{75}\right) = j\,0.15 \text{ p.u}$$

Reactance of generator G_2

$X_{p.u.(given)} = 0.1$ p.u., $MVA_{given} = 25$, $MVA_{new} = 75$, $kV_{given} = 11$, $kV_{new} = 11$

$$X_{p.u.(new)} = 0.1 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{75}{25}\right) = j\,0.3 \text{ p.u.}$$

Reactance of transformer T_1: (primary side)

$X_{p.u.(given)} = 0.1$ p.u., $MVA_{given} = 75$, $MVA_{new} = 75$, $kV_{given} = 11$, $kV_{new} = 11$

$$X_{p.u.(new)} = j\,0.1 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{75}{75}\right) = j\,0.1 \text{ p.u.}$$

Reactance of transformer T_2: (primary side)

$X_{p.u.(given)} = 0.08$ p.u., $MVA_{given} = 25$, $MVA_{new} = 75$, $kV_{given} = 11$, $kV_{new} = 11$

$$X_{p.u.(new)} = j\,0.08 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{75}{25}\right) = j\,0.24 \text{ p.u.}$$

Reactance of the transmission line

150 km long, actual reactance = 0.2 × 150 = 30 Ω

Base kV on HT side of transformer T_1

$$= \text{base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

Base kV on HT side of transformer $T_1 = 11 \times \frac{132}{11} = 132$ kV

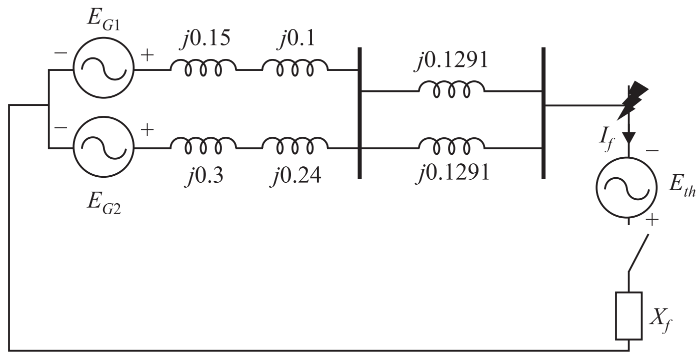
$kV_{new} = 132$ kV

$$\text{Base impedance} = \frac{(kV_{new})^2}{MVA_{new}} = \frac{132^2}{75} = 232.32\ \Omega$$

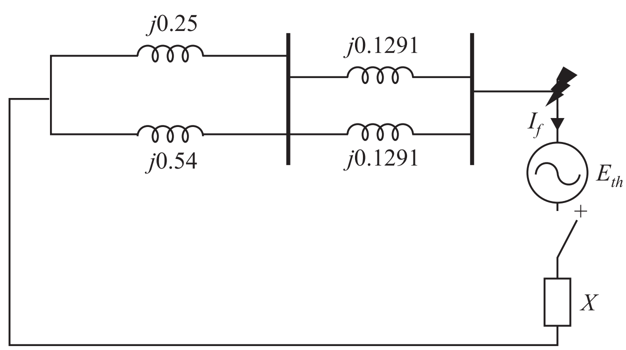
Per unit reactance of transmission line

$$= \frac{\text{actual reactance, }\Omega}{\text{base reactance, }\Omega} = \frac{30}{232.32} = j0.1291 \text{ p.u.}$$

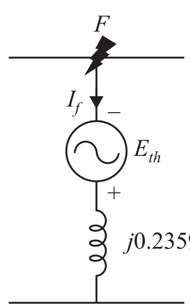
Prefault reactance diagram of Example 3.3



Thevenin reactance network of Example 3.3



Thevenin equivalent network of Example 3.3



Thevenin equivalent impedance, X_{th} is $(j0.25 \parallel j0.54) + (j0.1291 \parallel j0.1291)$

$$\therefore \qquad Z_{th} \text{ or } X_{th} = \frac{j0.25 \times j0.54}{j0.25 + j0.54} + \frac{j0.1291 \times j0.1291}{j0.1291 + j0.1291}$$

$$= j0.1708 + j0.065 = j0.2358 \text{ p.u.}$$

Per unit prefault voltage or Thevenin voltage, $E_{th} = 1\angle 0°$

Fault reactance or impedance, $X_f = 0$

Fault current

$$I_f = \frac{E_{th}}{j(X_{th} + X_f)} = \frac{1\angle 0°}{j0.2358} = -j4.241 \text{ p.u.}$$

$$\text{Base current} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\text{kV}_b} \times 10^3 = \frac{75}{\sqrt{3} \times 132} \times 10^3 = 328.04 \text{ A}$$

Actual value of fault current in kA = fault current in p.u.(I_f) × base current

Actual value of fault current in $|I_f|$ = 4.241 × 328.04 = 1391.22 A

Fault current supplied by each generator

$$\text{Base current for primary side of transformer} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\text{kV}_b} \times 10^3$$

$$= \frac{75}{\sqrt{3} \times 11} \times 10^3 = 3936.5 \text{ A}$$

Actual value of fault current = fault current in p.u.(I_f) × base current

Actual value of fault current = –j4.241 × 3936.5 = –j16,694.6 A

$$I_{G1} = \frac{jX_{g2}}{j(X_{g1} + X_{g2})} I_f = \frac{j0.54}{j(0.25 + 0.54)} \times -j16,694.6 = -j11,411.5$$

$$I_{G2} = \frac{jX_{g1}}{j(X_{g1} + X_{g2})} I_f = \frac{j0.25}{j(0.25 + 0.54)} \times -j16,694.6 = -j5283.10$$

EXAMPLE 3.4 A 25 MVA, 11 kV generator with 20% subtransient reactance is connected through a transformer to a bus which supplies four identical motors, as shown in Figure 3.16. The subtransient reactance X_d'' of each motor is 20% on a base of 5 MVA, 6.6 kV. The three-phase rating of the transformer is 25 MVA, 11/6.6 kV, with a leakage reactance of 10%. The bus voltage at the motors is 6.6 kV when a three-phase fault occurs at the point F. For the fault specified, calculate (i) the subtransient current in the fault, (ii) the subtransient current in breaker A and (iii) the momentary current in breaker A.

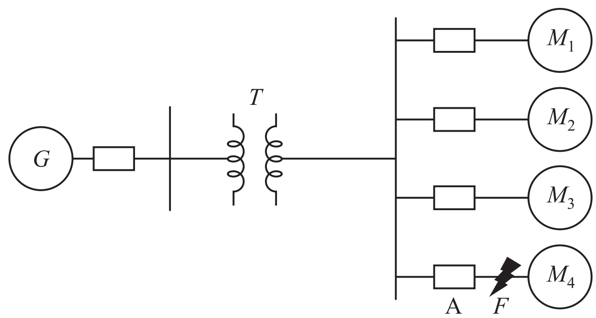


Figure 3.16 Single line diagram.

Solution:

Base MVA, MVA_{new} = 25 MVA
Base kV, kV_{new} = 11 kV

Reactance of generator G

$X_{p.u.(given)} = 0.2$ p.u., $MVA_{given} = 25$, $MVA_{new} = 25$, $kV_{given} = 11$, $kV_{new} = 11$

$$X_{p.u.(new)} = 0.2 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{25}{25}\right) = 0.2 \text{ p.u.}$$

Reactance of transformer T_1 (Primary side)

$X_{p.u.(given)} = 0.1$ p.u., $MVA_{given} = 25$, $MVA_{new} = 25$, $kV_{given} = 11$, $kV_{new} = 11$

$$X_{p.u.(new)} = j0.1 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{25}{25}\right) = j0.1 \text{ p.u.}$$

Reactance of motor

$X_{p.u.(given)} = 0.2$ p.u., $MVA_{given} = 5$, $MVA_{new} = 25$, $kV_{given} = 6.6$, $kV_{new} = ?$

Base kV on LT side of transformer T_1

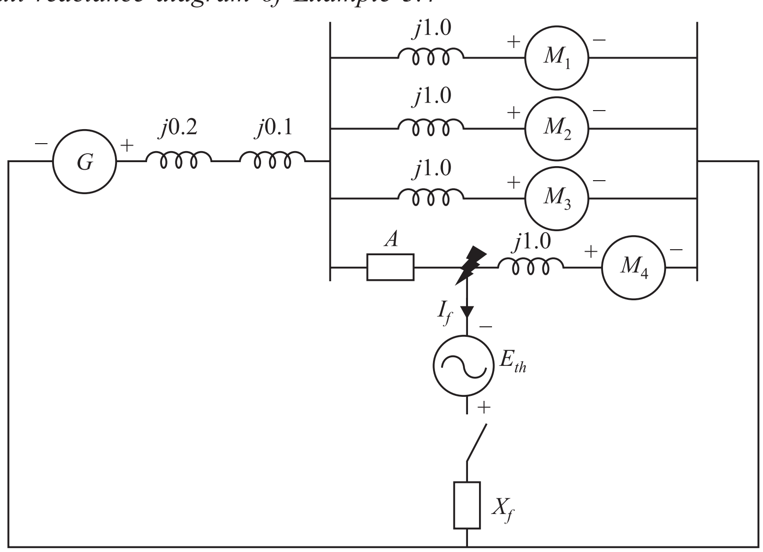
$$= \text{base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

Base kV on HT side of transformer $T_1 = 11 \times \dfrac{6.6}{11} = 6.6$ kV

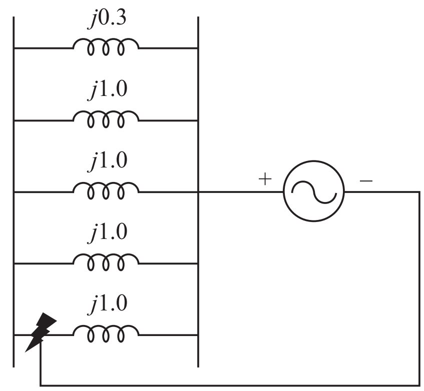
kV_{new} = 6.6 kV

$$X_{p.u.(new)} = j0.2 \times \left(\frac{6.6}{6.6}\right)^2 \times \left(\frac{25}{5}\right) = j1.0 \text{ p.u.}$$

Prefault reactance diagram of Example 3.4



(i) *Subtransient state:* The prefault reactance diagram is shown above. The system being initially on no load, therefore, the generator and the motor induced emfs are each equal to $1\angle 0°$ p.u. The prefault reactance diagram can be reduced to Thevenin equivalent network as shown below.



Thevenin equivalent impedance, X_{th} is

$$(j0.3 \parallel j1.0 \parallel j1.0 \parallel j1.0 \parallel j1.0)$$

$$\therefore \quad \frac{1}{Z_{th}} \text{ or } \frac{1}{X_{th}} = \frac{1}{j0.3} + \frac{1}{j1} + \frac{1}{j1} + \frac{1}{j1} + \frac{1}{j1}$$

$$\frac{1}{X_{th}} = -j3.3 - j1 - j1 - j1 - j1 = -j7.33 \text{ p.u.}$$

$$X_{th} = \frac{1}{-j7.33} = j0.1364$$

Per unit prefault voltage or Thevenin voltage, $E_{th} = 1\angle 0°$
Fault reactance or impedance, $X_f = 0$
Fault current

$$I_f = \frac{E_{th}}{j(X_{th} + X_f)} = \frac{1\angle 0°}{j0.1364} = -j7.33 \text{ p.u.}$$

$$\text{Base current} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\,\text{kV}_b} \times 10^3 = \frac{25}{\sqrt{3} \times 6.6} \times 10^3 = 2186.93 \text{ A}$$

Actual value of fault current = fault current in p.u.(I_f) × base current

Actual value of fault current $|I_f|$ = 7.33 × 2186.93
= 16030.2 A

(ii) Subtransient current in breaker A is supplied by the generator and motors M_1, M_2 and M_3. Therefore, the subtransient in breaker A may be written as.
Thevenin equivalent impedance, X_{th} is $(j0.3 \parallel j1.0 \parallel j1.0 \parallel j1.0)$

$$\therefore \qquad \frac{1}{Z_{th}} \text{ or } \frac{1}{X_{th}} = \frac{1}{j0.3} + \frac{1}{j1} + \frac{1}{j1} + \frac{1}{j1}$$

$$\frac{1}{X_{th}} = -j3.3 - j1 - j1 - j1 = -j6.33 \text{ p.u.}$$

$$X_{th} = \frac{1}{-j6.33} = j0.15798$$

Per unit prefault voltage or Thevenin voltage, $E_{th} = 1\angle 0°$

Fault reactance or impedance, $X_f = 0$

Fault current

$$I_f = \frac{E_{th}}{j(X_{th} + X_f)} = \frac{1\angle 0°}{j0.15798} = -j6.33 \text{ p.u.}$$

$$\text{Base current} = \frac{\text{MVA}_{\text{base}}}{\sqrt{3}\ \text{kV}_b} \times 10^3 = \frac{25}{\sqrt{3} \times 6.6} \times 10^3 = 2186.93 \text{ A}$$

Actual value of fault current = fault current in p.u.$(I_f) \times$ base current

Actual value of fault current $|I_f|$ = 6.33 × 2186.93

= 13843.08 A

(iii) For finding momentary current through breaker A, we must add the dc offset current to the symmetrical subtransient obtained above in part (ii). A factor 1.6 is taken into account for dc component.

Thus the momentary current through breaker A = 1.6 × 13843.08

= 22148.9 A

3.6 Fault Calculation Using Bus Impedance Matrix

The three-phase short-circuit fault current calculation used in the last chapter is not efficient and is not applicable to a large interconnected network.

Consider a sample n bus system network as shown in Figure 3.17. It is assumed that the system is operating under balanced condition and a per phase

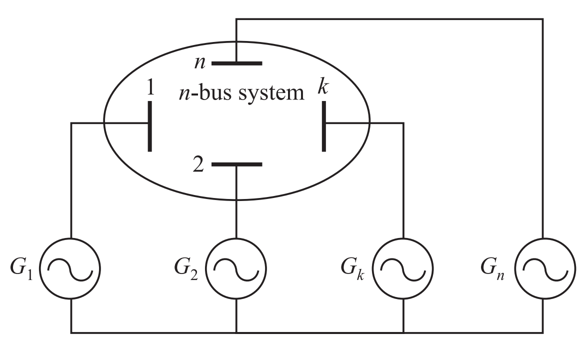


Figure 3.17 Sample n-bus system.

circuit model is used. A balanced three fault is to be applied at bus k through fault impedance.

The first step in the short-circuit study is to determine the prefault bus voltage and line current using the load flow study.

Prefault bus voltage can be defined as
$$V_{\text{bus}}^0 = \begin{bmatrix} V_1^0 \\ V_2^0 \\ \vdots \\ V_k^0 \\ \vdots \\ V_n^0 \end{bmatrix} \quad (3.33)$$

where, V_1^0, V_2^0, V_k^0 and V_n^0 are the prefault bus voltages.

Let bus k be the faulted bus and Z_f be the fault impedance. The postfault bus voltage vector is given by

$$V_{\text{bus}}^f = V_{\text{bus}}^0 + \Delta V \quad (3.34)$$

where ΔV is the change in bus voltage caused by the fault and is given by

$$\Delta V = \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_n \end{bmatrix} \quad (3.35)$$

Figure 3.18 shows the Thevenin's network of the system with generator replaced by transient/subtransient reactance with their emfs shorted.

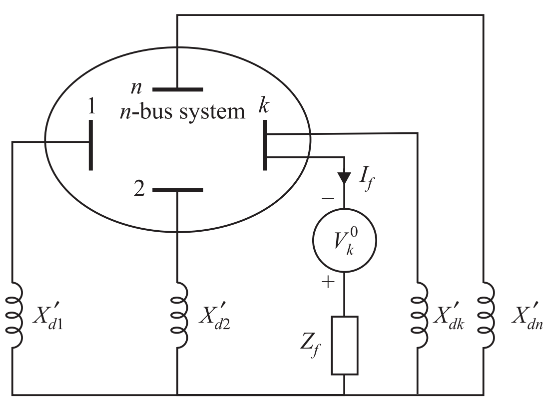


Figure 3.18 Changes in bus voltages caused by the fault.

In Figure 3.18, we excite the passive Thevenin network with $-V_k^0$ is in series with Z_f.

Now

$$\Delta V = Z_{\text{bus}} C_f \tag{3.36}$$

where $[Z_{\text{bus}}]$ is the bus impedance matrix of the passive thevenin network and is given by

$$Z_{\text{bus}} = \begin{bmatrix} Z_{11} & \cdots & Z_{1n} \\ \vdots & \ddots & \vdots \\ Z_{n1} & \cdots & Z_{nn} \end{bmatrix} \tag{3.37}$$

and C_f is the bus current injection vector. The network is injected with current $-I_f$ only at the *kth* bus, we have

$$C_f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_k = -I_f \\ \vdots \\ 0 \end{bmatrix} \tag{3.38}$$

From Eqs. (3.36) and (3.38)

$$\Delta V_k = -Z_{kk} I_f \tag{3.39}$$

The voltage at the *kth* bus under fault is

$$V_{k(f)} = V_k^0 + \Delta V_k = V_k^0 - Z_{kk} I_f \tag{3.40}$$

$$V_{k(f)} = Z_f I_f \tag{3.41}$$

$$Z_f I_f = V_k^0 - Z_{kk} I_f \tag{3.42}$$

$$I_f = \frac{V_k^0}{Z_{kk} + Z_f} \tag{3.43}$$

Using Eq. (3.39), at the *ith* bus ($k = i$)

$$\Delta V_i = -Z_{ii} I_f \tag{3.44}$$

Similarly from Eq. (3.40)

$$V_{i(f)} = V_i^0 + \Delta V_i = V_i^0 - Z_{ik} I_f \tag{3.45}$$

Substituting for I_f, the bus voltage during the fault at bus i becomes

$$V_{i(f)} = V_i^0 - \frac{Z_{ik}}{(Z_{kk} + Z_f)} V_k^0 \tag{3.46}$$

For $i = k$, Eq. (3.46) becomes

$$V_{k(f)} = V_i^0 - \frac{Z_{kk}}{(Z_{kk} + Z_f)} V_k^0$$

$$V_{k(f)} = \frac{Z_{kk}}{(Z_{kk} + Z_f)} V_k^0 \tag{3.47}$$

Note that V_i^0 are the prefault bus voltage and can be obtained from the load flow study. Z_{bus} matrix for the short-circuit study can be obtained by inversing Y_{bus} matrix. Also note that synchronous motors must be included in Z_{bus} formulation for the short-circuit study. However, in formulating short-circuit study network, load impedances are ignored, because these are very much larger than the impedances of generators and transmission lines.

Fault current flowing from bus i to bus j with impedance z_{ij} is given by

$$I_{ij(f)} = \frac{(V_{i(f)} - V_{j(f)})}{z_{ij}} = Y_{ij}(V_{i(f)} - V_{j(f)}) \tag{3.48}$$

Prefault generator current can be obtained by referring Figure 3.19(a).

Prefault generator = $P_{Gi} + jQ_{Gi}$

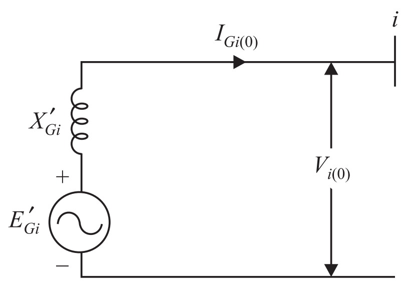
$$\therefore \qquad I_{Gi(0)} = \frac{P_{Gi} + jQ_{Gi}}{V_{i(0)}} \tag{3.49}$$

From Figure 3.19(a), we get

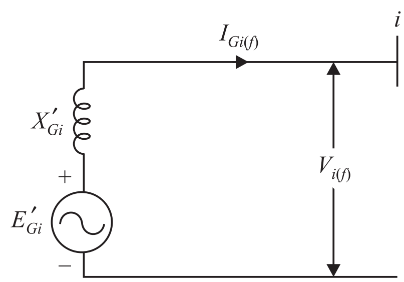
$$E'_{Gi} = V_{i(0)} + jX'_{Gi} I_{Gi(0)} \tag{3.50}$$

From the short-circuit study, $V_{i(f)}$ is obtained, from Figure 3.19(b)

$$I_{Gi(f)} = \frac{E'_{Gi} - V_{i(f)}}{jX'_{Gi}} \tag{3.51}$$



(a) Equivalent circuit of synchronous generator at prefault condition



(b) Equivalent circuit of synchronous generator at postfault condition

Figure 3.19

EXAMPLE 3.5 Consider the three bus system as shown in Figure 3.20. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common 100 MVA base. A three-phase fault with a fault impedance $Z_f = j0.15$ p.u. occurs at bus 3 using the bus impedance matrix method; calculate the fault current, the bus voltages, and the line current during the fault.

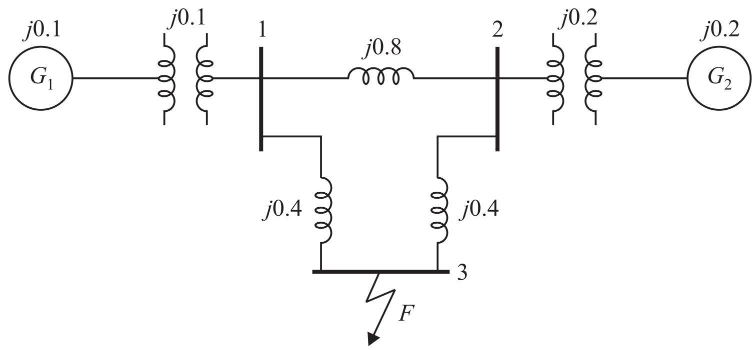


Figure 3.20 Single line diagram.

Solution:

Step 1 Determine the bus admittance matrix using the direct inspection method.

$$Y_{11} = \frac{1}{j0.2} + \frac{1}{j0.8} + \frac{1}{j0.4} = -j8.75$$

$$Y_{12} = Y_{21} = \frac{-1}{j0.8} = j1.25$$

$$Y_{13} = Y_{31} = \frac{-1}{j0.4} = j2.5$$

$$Y_{22} = \frac{1}{j0.4} + \frac{1}{j0.8} + \frac{1}{j0.4} = -j6.25$$

$$Y_{23} = Y_{32} = \frac{-1}{j0.4} = j2.5$$

$$Y_{33} = \frac{1}{j0.4} + \frac{1}{j0.4} = -j5.0$$

$$Y_{\text{bus}} = \begin{bmatrix} -j8.75 & j1.25 & j2.5 \\ j1.25 & -j6.25 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix}$$

Step 2 Determine the bus impedance matrix $Z_{\text{bus}} = \dfrac{1}{Y_{\text{bus}}}$.

$$Z_{\text{bus}} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}.$$

Step 3 Determine the fault current.

For a fault at bus 3 ($k = 3$) with fault impedance $Z_f = j0.15$ p.u.

Prefault voltages of all buses, $V_{1(0)} = V_{2(0)} = V_{3(0)} = 1.0$ p.u.

The fault current is

$$I_f = \frac{V_{k(0)}}{Z_{kk} + Z_f} = \frac{V_{3(0)}}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.15} = -j2.041 \text{ p.u.}$$

Step 4 Determine the bus voltages.

The bus voltages during the fault are:

$$V_{i(f)} = V_{i(0)} - Z_{ik}I_f$$

$$V_{1(f)} = V_{1(0)} - Z_{13}I_f = 1.0 - (j0.12) \times (-j0.2041) = 0.7551 \text{ p.u.}$$

$$V_{2(f)} = V_{2(0)} - Z_{23}I_f = 1.0 - (j0.16) \times (-j0.2041) = 0.6734 \text{ p.u.}$$

$$V_{3(f)} = V_{3(0)} - Z_{33}I_f = 1.0 - (j0.34) \times (-j0.2041) = 0.3060 \text{ p.u.}$$

Step 5 Determine the line current during the fault.

$$I_{ij(f)} = \frac{V_{i(f)} - V_{j(f)}}{z_{ij}}$$

$$I_{12(f)} = \frac{V_{1(f)} - V_{2(f)}}{z_{12}} = \frac{0.7551 - 0.6734}{j0.8} = -j0.102 \text{ p.u.}$$

$$I_{13(f)} = \frac{V_{1(f)} - V_{3(f)}}{z_{13}} = \frac{0.7551 - 0.3060}{j0.4} = -j1.122 \text{ p.u.}$$

$$I_{23(f)} = \frac{V_{2(f)} - V_{3(f)}}{z_{23}} = \frac{0.6734 - 0.3061}{j0.4} = -j0.918 \text{ p.u.}$$

EXAMPLE 3.6 The bus impedance matrix of a four-bus network with values in per unit is

$$Z_{\text{bus}} = j\begin{bmatrix} 0.15 & 0.08 & 0.04 & 0.07 \\ 0.08 & 0.15 & 0.06 & 0.09 \\ 0.04 & 0.06 & 0.13 & 0.05 \\ 0.07 & 0.09 & 0.05 & 0.12 \end{bmatrix}$$

The generators connected to buses 1 and 2 have their subtransient reactances included in Z_{bus}. If the prefault current is neglected, determine the subtransient current in per unit in the fault for a three-phase fault on bus 4. Assume the voltage at the fault is $1.0\angle 0°$ p.u. before the fault occurs. Find also the per unit current from generator 2, whose subtransient reactance is 0.2 p.u.

Solution: Fault at bus 4 ($k = 4$) with fault impedance, $Z_f = 0$ p.u.

Prefault voltages of all buses, $V_{1(0)} = V_{2(0)} = V_{3(0)} = V_{4(0)} = 1.0$ p.u.

The fault current is

$$I_f = \frac{V_{k(0)}}{Z_{kk} + Z_f} = \frac{V_{4(0)}}{Z_{44} + Z_f} = \frac{1.0}{j0.12} = -j8.33 \text{ p.u.}$$

$$V_{2(f)} = V_{2(0)} - Z_{24}I_f = 1.0 - (j0.09) \times (-j8.33) = 0.2503 \text{ p.u.}$$

Per unit current from generator 2,

$$I_{Gi(f)} = \frac{E'_{Gi} - V_{i(f)}}{jX'_{Gi}}$$

$$I_2 = \frac{1.0 - V_{2(f)}}{j0.2} = \frac{1.0 - 0.2503}{j0.2} = -j3.75 \text{ p.u.}$$

EXAMPLE 3.7 Consider the three bus network as shown in Figure 3.21. Determine the subtransient current in p.u. from generator 1 and in line 1–2 and voltages at buses 1 and 3 for three-phase fault on bus 2. Use the bus impedance matrix.

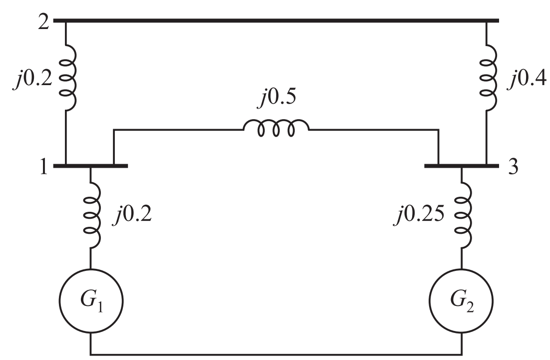


Figure 3.21 Three bus network for Example 3.7.

Solution:

Step 1 Determine the bus admittance matrix using the direct inspection method.

$$Y_{11} = \frac{1}{j0.2} + \frac{1}{j0.2} + \frac{1}{j0.5} = -j12$$

$$Y_{12} = Y_{21} = \frac{-1}{j0.2} = j5$$

$$Y_{13} = Y_{31} = \frac{-1}{j0.5} = j2$$

$$Y_{22} = \frac{1}{j0.2} + \frac{1}{j0.4} = -j7.5$$

$$Y_{23} = Y_{32} = \frac{-1}{j0.4} = j2.5$$

$$Y_{33} = \frac{1}{j0.25} + \frac{1}{j0.4} + \frac{1}{j0.5} = -j8.5$$

$$Y_{\text{bus}} = \begin{bmatrix} -j12 & j5 & j2 \\ j5 & -j7.5 & j2.5 \\ j2 & j2.5 & -j8.5 \end{bmatrix}$$

Step 2 Determine the bus impedance matrix $Z_{\text{bus}} = \dfrac{1}{Y_{\text{bus}}}$.

$$Z_{\text{bus}} = \begin{bmatrix} j0.1447 & j0.1195 & j0.0692 \\ j0.1195 & j0.2465 & j0.1006 \\ j0.0692 & j0.1006 & j0.1631 \end{bmatrix}.$$

Step 3 Determine the fault current.
For a fault at bus 2 (k = 2) with fault impedance Z_f = 0 p.u.
Prefault voltages of all buses, $V_{1(0)} = V_{2(0)} = V_{3(0)}$ = 1.0 p.u.
The fault current is

$$I_f = \frac{V_{k(0)}}{Z_{kk} + Z_f} = \frac{V_{2(0)}}{Z_{22} + Z_f} = \frac{1.0}{j0.2465} = -j4.057 \text{ p.u.}$$

Step 4 Determine the bus voltages.
Bus voltages during the fault are:

$$V_{i(f)} = V_{i(0)} - Z_{ik}I_f$$
$$V_{1(f)} = V_{1(0)} - Z_{12}I_f = 1.0 - (j0.1195) \times (-j4.057) = 0.515 \text{ p.u.}$$
$$V_{2(f)} = V_{2(0)} - Z_{22}I_f = 1.0 - (j0.2465) \times (-j4.057) = 5 \times 10^{-5} \text{ p.u.}$$
$$V_{3(f)} = V_{3(0)} - Z_{32}I_f = 1.0 - (j0.1006) \times (-j4.057) = 0.592 \text{ p.u.}$$

Step 5 Determine the line current during the fault.

$$I_{ij(f)} = \frac{V_{i(f)} - V_{j(f)}}{z_{ij}}$$

$$I_{12(f)} = \frac{(V_{1(f)} - V_{2(f)})}{z_{12}} = \frac{0.515 - 5 \times 10^{-5}}{j0.2} = -j2.575 \text{ p.u.}$$

$$I_{Gi(f)} = \frac{E'_{Gi} - V_{i(f)}}{jX'_{Gi}}$$

$$I_1 = \frac{1.0 - 0.515}{j0.2} = -j2.43 \text{ p.u.}$$

3.7 Algorithm for Formation of the Bus Impedance Matrix

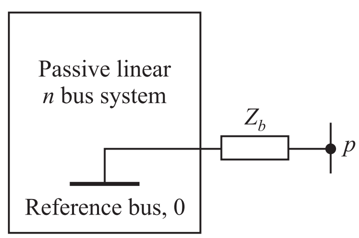
The bus impedance algorithm is a step by step procedure which proceeds branch by branch. The main advantage of this method is that any modification of the network elements does not require complete rebuilding of [Z_{bus}] matrix.

The bus impedance matrix can be build up starting with a single element and the process is continued until all the nodes and elements are included. Let us assume that the Z_{bus} matrix exists for a partial network having n buses and a reference bus is as shown in the figure below. It is proposed to add a new element one at a time to this network and get modified [Z_{bus}] matrix in the following four ways or four modifications.

Type 1 modification:	Add an element with impedance Z_b, connected between the reference node 0 and a new node p
Type 2 modification:	Add an element with impedance Z_b, connected between the existing node i and a new node p
Type 3 modification:	Add an element with impedance Z_b, connected between the existing node i and a reference node 0
Type 4 modification:	Add an element with impedance Z_b, connected between the existing nodes i and j

1. Add an element with impedance Z_b, connected between the reference node 0 and a new node p.

   In this case the addition of a new bus p to the reference node through impedance Z_b without a connection to any of the buses of the original network cannot alter the original bus voltage when a current is injected at the new bus.

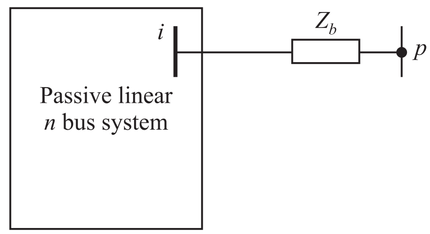
   

   The modified [Z_{bus}] matrix is given by

$$Z_{bus}^{new} = \left[\begin{array}{cccc|c} & & & & 0 \\ & & & & 0 \\ & Z_{bus}^{old} & & & \vdots \\ & & & & 0 \\ \hline 0 & 0 & \cdots & 0 & Z_b \end{array}\right] \tag{3.52}$$

2. Add an element with impedance Z_b, connected between the existing node i and a new node p.

   Consider impedance Z_b, connected between the existing node i and the new node p. The addition of bus will increase the order of the bus impedance matrix by one.



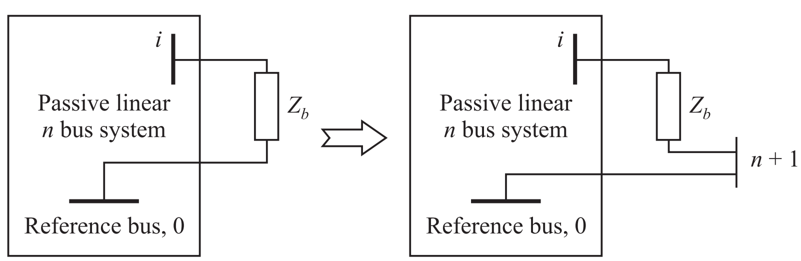
The modified $[Z_{\text{bus}}]$ matrix is given by

$$Z_{\text{bus}}^{\text{new}} = \left[\begin{array}{cccc|c} & & & & Z_{1i} \\ & & & & Z_{2i} \\ & Z_{\text{bus}}^{\text{old}} & & & \vdots \\ & & & & Z_{ni} \\ \hline Z_{i1} & Z_{i2} & \cdots & Z_{in} & Z_{ii} + Z_b \end{array}\right] \tag{3.53}$$

3. Add an element with impedance Z_b, connected between the existing node i and the reference node 0.

   To find the new $[Z_{\text{bus}}]$, the first step is to add an element in between the existing node i and a fictitious node $(n + 1)$ (instead of the reference node) and calculate the modified $[Z_{\text{bus}}]$ matrix of dimension $(n + 1) \times (n + 1)$ by using the matrix type 2 modification. The second step is to connect the fictitious node $(n + 1)$ by zero impedance link to the reference node whose voltage is zero.

   The new modified $[Z_{\text{bus}}]$ matrix of dimension $n \times n$ is obtained by applying Kron's reduction formula to the last row and column using the following relation.

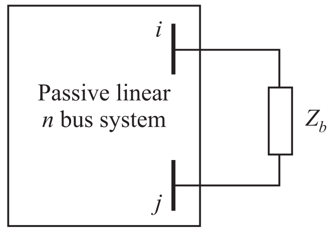


$$Z_{jk}^{\text{new}} = Z_{jk}^{\text{old}} - \frac{Z_{j(n+1)} \times Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \qquad j, k = 1, 2, ..., n \tag{3.54}$$

   Here the size of the matrix will not change because no new node is added.

4. Add an element with impedance Z, connected between the existing nodes i and j.

   Consider an element Z_b, connected between two existing nodes i and j, the new modified Z_{bus} matrix is given by



$$Z_{bus}^{new} = \left[\begin{array}{cccc|c} & & & & Z_{1j} - Z_{1i} \\ & & & & Z_{2j} - Z_{2i} \\ & Z_{bus}^{old} & & & \vdots \\ & & & & Z_{nj} - Z_{ni} \\ \hline Z_{j1} - Z_{i1} & Z_{j2} - Z_{i2} & \cdots & Z_{jn} - Z_{in} & Z_{ii} + Z_{jj} - 2Z_{ij} + Z_b \end{array}\right] \tag{3.55}$$

Now the size of the matrix becomes $(n + 1) \times (n + 1)$. The new modified $[Z_{bus}]$ matrix of dimension $(n \times n)$ is obtained by applying Kron's reduction formula to the last row and column using the following relation.

$$Z_{jk}^{new} = Z_{jk}^{old} - \frac{Z_{j(n+1)} \times Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \qquad j, k = 1, 2, ..., n$$

EXAMPLE 3.8 The impedance matrix $[Z_{bus}]$ as shown in Figure 3.22.

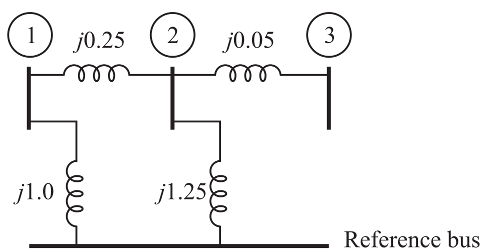
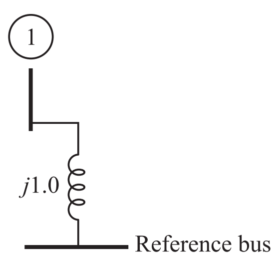


Figure 3.22

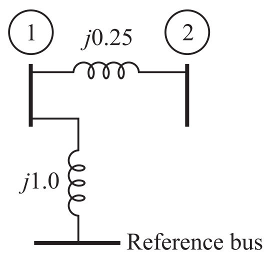
Solution:

1. Add an element with impedance $j1.0$, connected between the reference node 0 and a new node 1. (*Type* 1 *modification*)



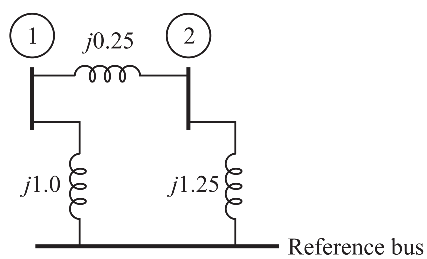
$$Z_{bus} = [j1.0]$$

2. Add an element with impedance $j0.25$, connected between the existing node 1 and a new node 2. (*Type* 2 *modification*)



$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.0 + j0.25 \end{bmatrix} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.25 \end{bmatrix}$$

3. Add an element with impedance $j1.25$, connected between the existing node 2 and the reference node 0. (*Type 3 modification*)



(a) To obtain [Z_{bus}] matrix use type 2 modification

$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j1.25 + j1.25 \end{bmatrix} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j2.50 \end{bmatrix}$$

(b) Now apply Kron's reduction formula

$$Z_{jk}^{new} = Z_{jk}^{old} - \frac{Z_{j(n+1)} \times Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$j, k = 1, 2$ and $(n + 1) = 0$ (reference node)

$j = 1; k = 1$ and $(n + 1) = 0$

$$Z_{11}^{new} = Z_{11}^{old} - \frac{Z_{10} \times Z_{01}}{Z_{00}} = j1.0 - \frac{j1.0 \times j1.0}{j2.50} = j0.6$$

$j = 1; k = 2$ and $(n + 1) = 0$

$$Z_{12}^{new} = Z_{12}^{old} - \frac{Z_{10} \times Z_{02}}{Z_{00}} = j1.0 - \frac{j1.0 \times j1.25}{j2.50} = j0.5$$

$j = 2;\ k = 1$ and $(n + 1) = 0$

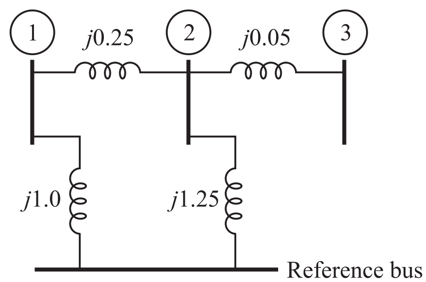
$$Z_{21}^{\text{new}} = Z_{21}^{\text{old}} - \frac{Z_{20} \times Z_{01}}{Z_{00}} = j1.0 - \frac{j1.25 \times j1.0}{j2.50} = j0.5$$

$j = 2;\ k = 2$ and $(n + 1) = 0$

$$Z_{22}^{\text{new}} = Z_{22}^{\text{old}} - \frac{Z_{20} \times Z_{02}}{Z_{00}} = j1.25 - \frac{j1.25 \times j1.25}{j2.50} = j0.625$$

$$Z_{\text{bus}} = \begin{bmatrix} j0.6 & j0.5 \\ j0.5 & j0.625 \end{bmatrix}$$

4. Add an element with impedance $j0.05$, connected between the existing node 2 and a new node 3. (*Type* 2 *modification*)



$$Z_{\text{bus}} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.625 + j0.05 \end{bmatrix}$$

$$Z_{\text{bus}} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.675 \end{bmatrix}$$

EXAMPLE 3.9 Obtain the bus impedance matrix [Z_{bus}] as shown in Figure 3.23 using Z_{bus} building algorithm.

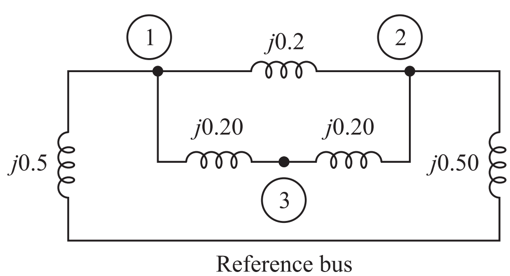
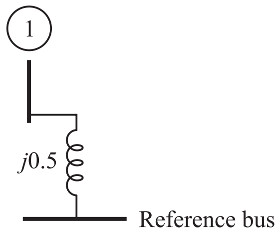


Figure 3.23

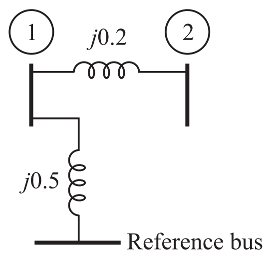
Solution:

1. Add an element with impedance $j0.5$, connected between the reference node 0 and a new node 1. (*Type* 1 *modification*)



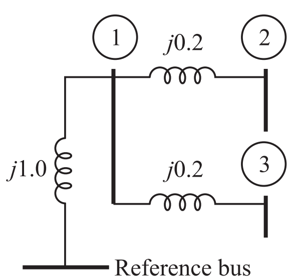
$$Z_{bus} = [j0.5]$$

2. Add an element with impedance $j0.2$, connected between the existing node 1 and a new node 2. (*Type* 2 *modification*)



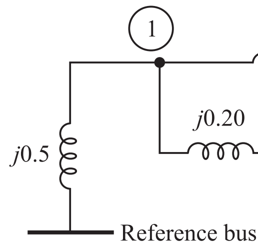
$$Z_{bus} = \begin{bmatrix} j0.5 & j0.5 \\ j0.5 & j0.5 + j0.2 \end{bmatrix} = \begin{bmatrix} j0.5 & j0.5 \\ j0.5 & j0.7 \end{bmatrix}$$

3. Add an element with impedance $j0.2$, connected between the existing node 1 and a new node 3. (*Type* 2 *modification*)



$$Z_{bus} = \begin{bmatrix} j0.5 & j0.5 & j0.5 \\ j0.5 & j0.7 & j0.5 \\ j0.5 & j0.5 & j0.5 + j0.2 \end{bmatrix} = \begin{bmatrix} j0.5 & j0.5 & j0.5 \\ j0.5 & j0.7 & j0.5 \\ j0.5 & j0.5 & j0.7 \end{bmatrix}$$

4. Add an element with impedance $j0.2$, connected between the existing nodes 2 and 3. (*Type* 4 *modification*)



$$Z_{bus}^{new} = \left[\begin{array}{cccc|c} & & & & Z_{1j} - Z_{1i} \\ & & & & Z_{2j} - Z_{2i} \\ & & Z_{bus}^{old} & & \vdots \\ & & & & Z_{nj} - Z_{ni} \\ \hline Z_{j1} - Z_{i1} & Z_{j2} - Z_{i2} & \cdots & Z_{jn} - Z_{in} & Z_{ii} + Z_{jj} - 2Z_{ij} + Z_b \end{array}\right]$$

$i = 2;\ j = 3$

$$Z_{bus}^{new} = \left[\begin{array}{cccc|c} & & & & Z_{13} - Z_{12} \\ & & & & Z_{23} - Z_{22} \\ & & Z_{bus}^{old} & & \vdots \\ & & & & Z_{33} - Z_{32} \\ \hline Z_{31} - Z_{21} & Z_{32} - Z_{22} & \cdots & Z_{33} - Z_{23} & Z_{22} + Z_{33} - 2Z_{23} + Z_b \end{array}\right]$$

$$Z_{bus} = \begin{bmatrix} j0.5 & j0.5 & j0.5 & j0.5 - j0.5 \\ j0.5 & j0.7 & j0.5 & j0.5 - j0.7 \\ j0.5 & j0.5 & j0.7 & j0.7 - j0.5 \\ j0.5 - j0.5 & j0.5 - j0.7 & j0.7 - j0.5 & j0.7 + j0.7 - 2 \times j0.5 + j0.2 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.5 & j0.5 & j0.5 & 0 \\ j0.5 & j0.7 & j0.5 & -j0.2 \\ j0.5 & j0.5 & j0.7 & j0.2 \\ 0 & -j0.2 & j0.2 & j0.6 \end{bmatrix}$$

Now apply Kron's reduction formula

$$Z_{jk}^{new} = Z_{jk}^{old} - \frac{Z_{j(n+1)} \times Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

$j, k = 1, 2, 3$ and $(n + 1) = 0$ (reference node)

$j = 1;\ k = 1$ and $(n + 1) = 0$

$$Z_{11}^{new} = Z_{11}^{old} - \frac{Z_{10} \times Z_{01}}{Z_{00}} = j0.5 - \frac{0 \times 0}{j0.6} = j0.5$$

$j = 1;\ k = 2$ and $(n + 1) = 0$

$$Z_{12}^{\text{new}} = Z_{12}^{\text{old}} - \frac{Z_{10} \times Z_{02}}{Z_{00}} = j0.5 - \frac{0 \times (-j0.2)}{j0.6} = j0.5$$

$j = 1;\ k = 3$ and $(n + 1) = 0$

$$Z_{13}^{\text{new}} = Z_{13}^{\text{old}} - \frac{Z_{10} \times Z_{03}}{Z_{00}} = j0.5 - \frac{0 \times (j0.2)}{j0.6} = j0.5$$

$j = 2;\ k = 1$ and $(n + 1) = 0$

$$Z_{21}^{\text{new}} = Z_{21}^{\text{old}} - \frac{Z_{20} \times Z_{01}}{Z_{00}} = j0.5 - \frac{(-j0.2) \times 0}{j0.6} = j0.5$$

$j = 2;\ k = 2$ and $(n + 1) = 0$

$$Z_{22}^{\text{new}} = Z_{22}^{\text{old}} - \frac{Z_{20} \times Z_{02}}{Z_{00}} = j0.7 - \frac{(-j0.2) \times (-j0.2)}{j0.6} = j0.633$$

$j = 2;\ k = 3$ and $(n + 1) = 0$

$$Z_{23}^{\text{new}} = Z_{23}^{\text{old}} - \frac{Z_{20} \times Z_{03}}{Z_{00}} = j0.5 - \frac{(-j0.2) \times (j0.2)}{j0.6} = j0.567$$

$j = 3;\ k = 1$ and $(n + 1) = 0$

$$Z_{31}^{\text{new}} = Z_{31}^{\text{old}} - \frac{Z_{30} \times Z_{01}}{Z_{00}} = j0.5 - \frac{(j0.2) \times 0}{j0.6} = j0.5$$

$j = 3;\ k = 2$ and $(n + 1) = 0$

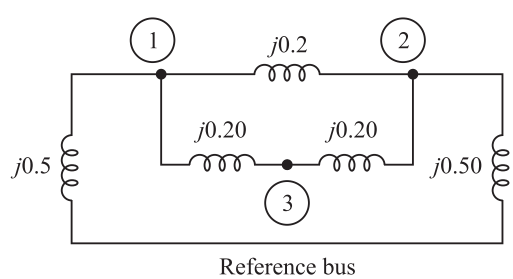
$$Z_{32}^{\text{new}} = Z_{32}^{\text{old}} - \frac{Z_{30} \times Z_{02}}{Z_{00}} = j0.5 - \frac{(j0.2) \times (-j0.2)}{j0.6} = j0.567$$

$j = 3;\ k = 3$ and $(n + 1) = 0$

$$Z_{33}^{\text{new}} = Z_{33}^{\text{old}} - \frac{Z_{30} \times Z_{03}}{Z_{00}} = j0.7 - \frac{(j0.2) \times (j0.2)}{j0.6} = j0.633$$

$$Z_{\text{bus}} = \begin{bmatrix} j0.5 & j0.5 & j0.5 \\ j0.5 & j0.633 & j0.567 \\ j0.5 & j0.567 & j0.633 \end{bmatrix}$$

6. Add an element with impedance $j0.5$, connected between the existing node 2 and the reference node 0. (*Type* 3 *modification*)



(a) To obtain $[Z_{bus}]$ matrix use type 2 modification

$$Z_{bus} = \begin{bmatrix} j0.5 & j0.5 & j0.5 & j0.5 \\ j0.5 & j0.633 & j0.567 & j0.633 \\ j0.5 & j0.567 & j0.633 & j0.567 \\ j0.5 & j0.633 & j0.567 & j0.633 + j0.5 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.5 & j0.5 & j0.5 & j0.5 \\ j0.5 & j0.633 & j0.567 & j0.633 \\ j0.5 & j0.567 & j0.633 & j0.567 \\ j0.5 & j0.633 & j0.567 & j1.133 \end{bmatrix}$$

(b) Now apply Kron's reduction formula

$$Z_{jk}^{new} = Z_{jk}^{old} - \frac{Z_{j(n+1)} \times Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

j, k = 1, 2, 3 and $(n + 1) = 0$ (reference node)

$j = 1$; $k = 1$ and $(n + 1) = 0$

$$Z_{11}^{new} = Z_{11}^{old} - \frac{Z_{10} \times Z_{01}}{Z_{00}} = j0.5 - \frac{j0.5 \times j0.5}{j1.133} = j0.2793$$

$j = 1$; $k = 2$ and $(n + 1) = 0$

$$Z_{12}^{new} = Z_{12}^{old} - \frac{Z_{10} \times Z_{02}}{Z_{00}} = j0.5 - \frac{j0.5 \times (j0.633)}{j1.133} = j0.2206$$

$j = 1$; $k = 3$ and $(n + 1) = 0$

$$Z_{13}^{new} = Z_{13}^{old} - \frac{Z_{10} \times Z_{03}}{Z_{00}} = j0.5 - \frac{j0.5 \times (j0.567)}{j1.133} = j0.2498$$

$j = 2$; $k = 1$ and $(n + 1) = 0$

$$Z_{21}^{new} = Z_{21}^{old} - \frac{Z_{20} \times Z_{01}}{Z_{00}} = j0.5 - \frac{(j0.633) \times j0.5}{j1.133} = j0.2206$$

$j = 2$; $k = 2$ and $(n + 1) = 0$

$$Z_{22}^{new} = Z_{22}^{old} - \frac{Z_{20} \times Z_{02}}{Z_{00}}$$

$$= j0.633 - \frac{(j0.633) \times (j0.633)}{j1.133} = j0.2793$$

$j = 2$; $k = 3$ and $(n + 1) = 0$

$$Z_{23}^{new} = Z_{23}^{old} - \frac{Z_{20} \times Z_{03}}{Z_{00}}$$

$$= j0.567 - \frac{(j0.633) \times (j0.567)}{j1.133} = j0.2502$$

$j = 3;\ k = 1$ and $(n + 1) = 0$

$$Z_{31}^{\text{new}} = Z_{31}^{\text{old}} - \frac{Z_{30} \times Z_{01}}{Z_{00}} = j0.5 - \frac{(j0.567) \times (j0.5)}{j1.133} = j0.2498$$

$j = 3;\ k = 2$ and $(n + 1) = 0$

$$Z_{32}^{\text{new}} = Z_{32}^{\text{old}} - \frac{Z_{30} \times Z_{02}}{Z_{00}}$$

$$= j0.567 - \frac{(j0.567) \times (j0.633)}{j1.133} = j0.2502$$

$j = 3;\ k = 3$ and $(n + 1) = 0$

$$Z_{33}^{\text{new}} = Z_{33}^{\text{old}} - \frac{Z_{30} \times Z_{03}}{Z_{00}}$$

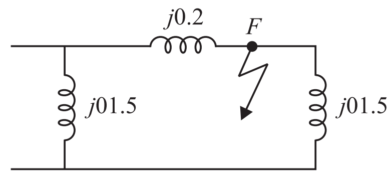
$$= j0.633 - \frac{(j0.567) \times (j0.567)}{j1.133} = j0.3492$$

$$Z_{\text{bus}} = \begin{bmatrix} j0.2793 & j0.2206 & j0.2498 \\ j0.2206 & j0.2793 & j0.2502 \\ j0.2498 & j0.2502 & j0.3492 \end{bmatrix}$$

Review Questions

Part-A

1. What is meant by a fault?
2. Why do faults occur in a power system?
3. List the various types of faults.
4. Write the relative frequency of occurrence of various types of faults.
5. State and explain the symmetrical fault or balanced three-phase fault.
6. What is the need for short-circuit studies or fault analysis?
7. What is bolted fault or solid fault?
8. What is the reason for transients during short-circuit?
9. What is meant by doubling effect?
10. Define dc offset current.
11. What is synchronous reactance or steady state condition reactance?
12. What is subtransient reactance?
13. What is transient reactance?
14. Define the short-circuit capacity of power system or fault level.
15. Find the fault current as given in the figure, if the prefault voltage at the fault point is 0.97 p.u.?

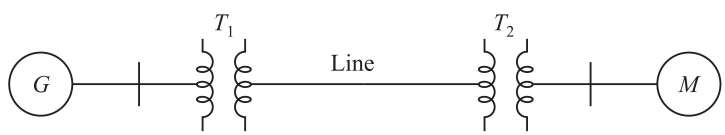


16. What is the bus impedance matrix?

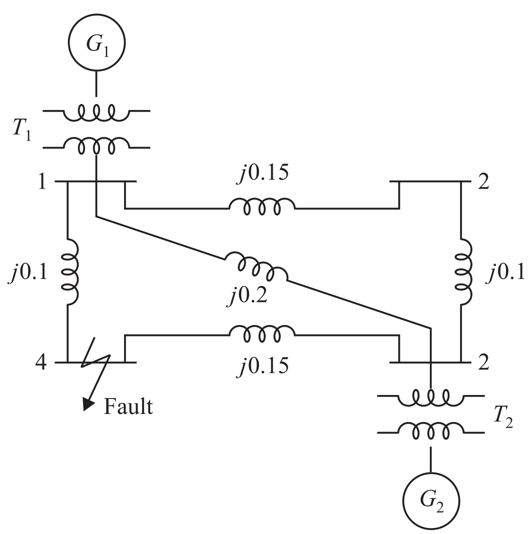
17. Give the methods available for forming the bus impedance matrix.

Part-B

1. A synchronous generator and a synchronous motor each rated 20 MVA, 12.66 kV having 15% reactance are connected through transformers and a line as shown in the figure below. The transformers are rated 20 MVA, 12.66/66 kV and 66/12.66 kV with leakage reactance of 10% each. The line has a reactance of 8% on base of 20 MVA, 66 kV. The motor is drawing 10 MW at 0.8 leading power factors and a terminal voltage 11 kV when symmetrical three-phase fault occurs at the motors terminals. Determine the generator and motor currents. Also determine the fault current.



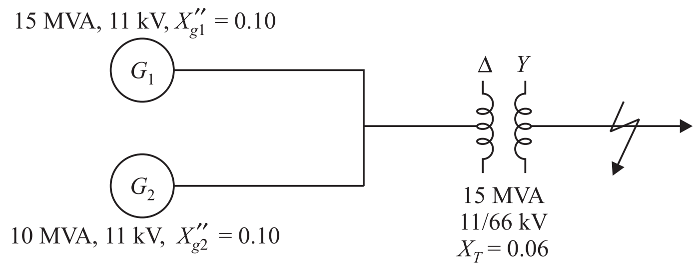
2. A 4-bus sample power system is shown in the figure. Perform the short-circuit analysis for a three-phase solid fault on bus 4. Data are given below.



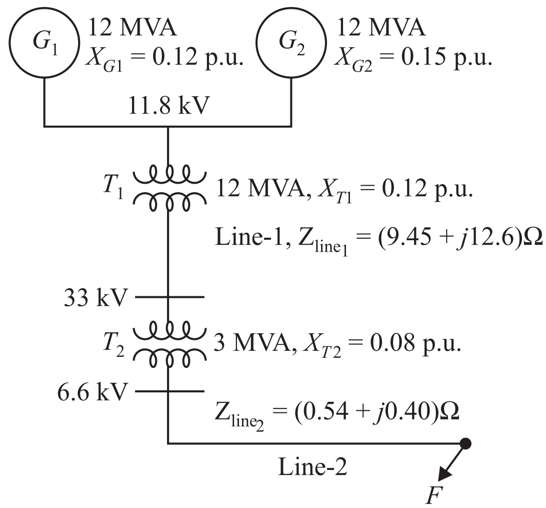
G_1: 11.2 kV, 100 MVA, X = 0.08 p.u.
G_2: 11.2 kV, 100 MVA, X = 0.08 p.u.
T_1: 11/110 kV, 100 MVA, X = 0.06 p.u.
T_2: 11/110 kV, 100 MVA, X = 0.06 p.u.

Assume the prefault voltage is 1.0 p.u. and the prefault current is zero.

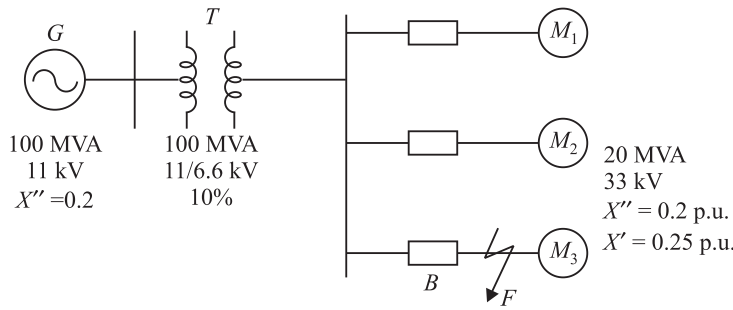
3. Two generators G_1 and G_2 are rated 15 MVA, 11 kV and 10 MVA, 11 kV respectively. The generators are connected to a transformer as shown in the figure. Calculate the subtransient current in each generator when a three-phase fault occurs on the high voltage side of the transformer.



4. A radial power system network is shown in the figure below. A three-phase balanced fault occurs at F. Determine the fault current and the line voltage at 11.8 kV bus under the fault condition.



5. A 100 MVA, 11 kV generator with X'' = 0.20 p.u. is connected through a transformer to a bus bar that supplies three identical motors as shown in the figure and each motor has X'' = 0.20 p.u and X' = 0.25 p.u. on a base of 20 MVA, 33 kV. The bus voltage at the motors is 33 kV when a three-phase balanced fault occurs at the point F. Calculate



(a) the subtransient current in the fault.
(b) the subtransient current in the circuit breaker B.
(c) the momentary current in the circuit breaker B.
(d) the current to be interrupted by circuit breaker B in (i) 2 cycles, (ii) 3 cycles, (iii) 5 cycles and (iv) 8 cycles.

6. Determine the impedance matrix Z_{bus} as shown in the figure below.